## MOVEMENT (II)

## OBJECTIVES

This unit continues with the study of movement started in Movement (I).

The aim is for a deeper understanding of the terms:
average acceleration vector, intrinsic components of acceleration.

More information is given about the vectorial description of accelerated movements.

Problematic situations are described which help you to understand rectilinear and circular, uniform and accelerated movements. Their graphic representations play an important role in the understanding of these movements.

The Teaching Unit ends with the study of simple movements like ascending and descending, and the crossing of moving objects.

You will find the information needed to begin to study this Teaching Unit in:

## Kinematics (2nd year) Rectilinear movements (4th year) Uniform circular

 movement (4th year)Trajectory and displacement (4th year)

### 1.1 What is acceleration?

When a body changes its velocity, it accelerates. When the velocity decreases, we usually say that it brakes.

Acceleration is represented by a vector, so that to completely define the acceleration of a body you have to specify its module, direction and orientation.

The module of acceleration gives us an idea of how quickly its velocity varies. If a body accelerates at $2 \mathrm{~m} / \mathrm{s}^{2}$ it means that its velocity increases by $2 \mathrm{~m} / \mathrm{s}$ every second.

The sign for acceleration depends on the Reference System we choose. So the acceleration of falling bodies (the acceleration of gravity or $g=9,8 \mathrm{~m} / \mathrm{s}^{2}$ ) can be written with a positive or negative sign: you decide which R.S. you choose.

To understand what acceleration is in more detail, it has to be strictly defined.

### 1.2 Average acceleration and instantaneous acceleration

Average acceleration is defined in the same way as average velocity. At the same time as you calculate the average velocity between two points, you can practise by observing average acceleration. If you choose shorter and shorter time intervals you will come close to the figure for instantaneous magnitudes.


VISUAL: This visual represents a body in circular motion. You may modify the radius of the movement, its speed and the time-scale that you wish to examine. Once this is done, you can see the average speed and average acceleration in smaller and smaller time intervals.

A1: Modify the values of $R$, the speed and $T$ and observe the changes in $r$ and $v$. Note the values of the vectors in the lower right window.

A2: Use fixed values of $R$, the speed and $T$, and select smaller and smaller time intervals. Observe the values of vm and am .

A3: Observe the fact that the average acceleration tends towards a definite value as the time intervals become shorter: this is the value of the instantaneous acceleration at a given time.

### 1.3 Intrinsic components of acceleration (I)

Instantaneous acceleration is a vector, so it can be split up into two perpendicular vectors in such a way that their sum is instantaneous velocity.

These vectors are:

- one tangential to the trajectory: tangential acceleration
- another perpendicular to tangential acceleration: normal or centripetal acceleration

Why is instantaneous acceleration split into two perpendicular vectors, one tangential to the trajectory at each point and another perpendicular?

In fact, it is a mathematical necessity which you may not yet understand, which is that acceleration is the derivative of the velocity vector with respect to time.

If you want to extend your knowledge about this, click
http://www.sc.ehu.es/sbweb/fisica/cinematica/curvilineo/curvilineo.htm

### 1.4 Intrinsic components of acceleration (II)

Use a complex movement to find out what the intrinsic components of acceleration are.


VISUAL: This visual represents a complex accelerated curvilinear movement. You can modify the values with two controls: R and speed. The total acceleration and its intrinsic components are represented for each point on the curve. Use the zoom to see these vectors clearly.

A1: Start the visual and stop it. You can see that the total acceleration is equal to the sum of its intrinsic components at each point. Let the animation continue and stop it again. Observe the change in the values along the curve.

A2: Try to answer the following questions: Is tangential acceleration always tangent to the trajectory? What points is the normal acceleration directed towards?

## Intrinsic components of acceleration (III)

Movements can be classified as to whether they show or don't show acceleration and what the intrinsic components of the latter are. Therefore,

| Intrinsic <br> components <br> of <br> acceleration | Rectilinear M. |  | Curvilinear M. |  | Other <br> (urvilinear |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{t}$ | 0 | UARM | UCM | UACM |  |
| $a_{n}$ | 0 | 0 | \# 0 and <br> constant | \# 0 and <br> constant | \# 0 |

## 2.1 s/t, v/t and a/t graphs

$\mathrm{s} / \mathrm{t}, \mathrm{v} / \mathrm{t}$ and $\mathrm{a} / \mathrm{t}$ graphs are extremely useful when studying movements.


A1: See if the initial graphs correspond to Uniform Movements.

A2: Change the value of acceleration and watch the variation in the s/t and v/t graphs.
A3: Change the values with the controls and select the maximum and minimum values for acceleration and velocity. You will observe great differences.

### 2.2 The meaning of the v/t graph

The area under the v/t graph changes as the movement ocurrs. Its value always coincides with the space covered by the moving object.


A1: Modify the values of $v$ and observe the value of the area of the trapezium (rectangle plus triangle) that is formed as time goes by. Remember that the area of the triangle is equal to base*height/2; in this case the base is equal to $t$ and the height is equal to $\mathrm{v}-\mathrm{v} 0$.

### 3.1 Angular magnitudes in circular movements

If a moving body has a circular movement, it is easier to express its speed by counting the number of revolutions it makes per unit of time than the metres it covers per unit of time. This is why in circular movements we use terms like: revolutions per minute, radians per second or revolutions per second.

Knowing how many times it revolves per second or per minute gives us an idea of how quickly it is moving.

We use the word "revolution" to express when an object completes a circuit so that the speed of a UCM is usually expressed in: r.p.m. ( revolutions per minute)

We also use the term r.p.s. (revolutions per second). Angular velocity is the term for the radians it covers in a second.

## Practise by answering the following questions

How long does the second hand of a clock take to make a complete circle? What does it mean when we say that the hard disk in a computer turns at 5400 r.p.m.?

How many times per hour does the minute hand circle the clock? (r.p.m.) How many times does the second hand circle a clock in one second in r.p.s.? What is the period of each of the hands of a clock?

Another simple way to say how quickly a UCM is moving is to express how long it takes to complete a revolution; this magnitude is also called period.

You can revise all these terms in Uniform circular movement 4th year

### 3.1 Angular magnitudes in circular movements

It is important to know the correspondence between radians and sexagesimal degrees to be able to express angular velocity, as this is measured in radians per second.


Sexagesimal: A circumference has 360 sexagesimal degrees.
A1: Drag the green dot and find out the values in radians for different angles.
A2: With a calculator, check that the angle in radians is equal to the length of arc divided by the radius.

E1: Calculate the value in radians of the following angles: 90, 180, 270 and 360 sexagesimal degrees.

E2: It is common to write angles in terms of pi (pi/4, pi/2, pi, 3/2(pi) and 2pi) instead of writing out the number of radians. Calculate the aforementioned angles in sexagesimal degrees.

### 3.2 Angular magnitudes are related to linear ones (I)

A body with a circular movement covers a space (s) which can be measured in metres: linear space or distance covered, and an angle . $\phi$ ) which is measured in radians: angular space. These two ways of describing displacement are related; the radius of the movement is decisive in this relation. Observe that at any moment the length of the arc is related to the space by $s=\phi^{*} r$


A1: With a calculator, check that the length of arc is always equal to the angle (measured in radians) times the radius.

Why?: If a circumference covers an angle of 2 pi radians and its length is equal to 2 pi*r, then a radian is equal to an arc of length equal to one radius.

### 3.2 Angular magnitudes are related to linear ones (II)

A body with a circular movement has angular velocity: $w$ and also linear velocity. Angular velocity $\omega$ is expressed in radians per second, linear velocity, v , in $\mathrm{m} / \mathrm{s}$. The relation between these two velocities is $\mathrm{v}=\omega^{*} \mathrm{r}$


VISUAL: Every time the visual is started, a different uniform circular movement is shown. Every point of the moving segment has a linear velocity.

A1: Check that the angular velocity of the UCM in the visual is always equal to the angular distance (in radians) divided by time.

A2: Check that the linear velocity of the UCM in the visual is equal to the angular velocity times the radius.

### 3.3 Graphs of circular movements

$\mathrm{s} / \mathrm{t}, \mathrm{v} / \mathrm{t}$ and a/t graphs of circular movements are the same as those for rectilinear movement, observe how when the magnitude of the two circular movements changes their graphs also change.

| A1 |  | A2 | A3 |  |  |  | config |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-7$ |  | $1$  |  | $2$ |  |  |  |  |
|  |  |  |  | leration $\square$ | (in |  |  |  |
| $\begin{aligned} & \mathrm{s} 1=15.8 \mathrm{~m} \\ & \mathrm{~s} 2=33.0 \mathrm{~m} \end{aligned}$ | (s) | time $\begin{aligned} & \mathrm{vl}=2.0 \mathrm{~m} / \mathrm{s} \\ & \mathrm{v} 2=7.9 \mathrm{~m} / \mathrm{s} \end{aligned}$ | (s) | $\begin{aligned} & \mathrm{al}=0.0 \\ & \mathrm{a} 2=1.0 \end{aligned}$ | (in/ <br> (道/ | $\begin{aligned} & \text { ime } \\ & s_{2}^{2} \\ & s^{2} \end{aligned}$ | ) |  |
| init vl ${ }^{\text {¢ }}$, 2.0 | v2 | $\stackrel{\Delta}{\square} 2.0$ radiusl $\frac{\Delta}{\nabla}$ | 1.0 |  | 1.0 | clear | 144 | II |

A1: See if the initial graphs correspond to a uniform movement.
A2: Change the value of the acceleration of one of the objects and watch the change in the $\mathrm{s} / \mathrm{t}$ and $\mathrm{v} / \mathrm{t}$ graphs.

A3: Change the values with the controls, choose the maximum and minimum values for the speed and acceleration. You will see great differences.

### 4.1 Freely falling bodies

A body falls in a vacuum from a height $h$. What type of movement does it show? What are the height/time and velocity/time graphs like?


VISUAL: Initially, the object is dropped. Space is measured from the ground.
A1: What is the meaning of each of the vectors represented? How does each of them vary?

A2: Drop the body from different heights, without an initial speed, and note down the initial height and the time it takes the object to reach the ground. Are the time intervals proportional to the heights? Are the velocities of the object when it reaches the ground proportional to the time intervals? Why are the dots on the trajectory more separated the longer the fall?

A3: Choose a fixed height and throw the object downwards at different speeds. Why are the $\mathrm{h} / \mathrm{t}$ graphs like the $\mathrm{v} / \mathrm{t}$ graphs? What is the difference between them? What would you say about the speed at which the object reaches the ground?

A4: Why are there negative values of the speed if the movement is uniformly accelerated?

### 4.2 Ascending and descending movement

A body is thrown vertically upwards. How does its velocity vary along its trajectory? How does its acceleration vary? What are its h/t and v/t graphs like?


VISUAL: The body is dropped, initially. If you change the values of v , you can make it go up initially.

A1: On the h/t graph, try to get the following graphs simultaneously: The object falling from a height under 5 m a)without an initial speed; b)with an initial speed +v . Don't clear after each experiment. Compare the two graphs: What are the similarities? What are the differences?

A2: On the $\mathrm{h} / \mathrm{t}$ graph, try to get the following graphs simultaneously: The object falling from a height under 5 m a)without an initial speed; b)with an initial speed $+\mathrm{v}, \mathrm{c}$ )with an initial speed $-v$. Don't clear after each experiment. Compare the three graphs: What are the similarities? What are the differences?

A3: Throw the object upwards from the ground and observe its speed when it reaches the ground again. Repeat the experiment for different speeds.

A4: Throw the object upwards from a fixed height $h$. Observe its speed when it reaches the ground. Without clearing the visual, drop the object from the same height. What is its speed when it reaches the ground? Compare the v/t graphs: What is the value of the separation between them on the coordinate axis?

### 4.3 The meeting of moving bodies

The graphs can help us to discover when and where two bodies which are moving in the same direction will meet.


VISUAL: You can change the initial position, initial speed and acceleration of both vehicles. When and where will they meet? Several screens are simulated in the visual, so the meeting point will not always be the real meeting point, only the first time they meet will the point correspond to the real meeting point.

A1: Move each vehicle to one of the edges of the visual. Set the acceleration of both vehicles to 0 . If the vehicle on the right has a negative speed and the one on the left has a positive speed, then they will meet on the screen. Modify the initial conditions to make them meet closer to one edge or the other. What graph shows where the vehicles will meet?

A2: Give one of the vehicles a head start. Change the accelerations so that they meet when the velocities are in the same direction and when they are in opposite directions.

## Evaluation

If after studying this unit, you have managed to:

Get to know and use the terms average acceleration vector, intrinsic components of acceleration.

Vectorially describe accelerated movements, knowing where to situate the velocity and acceleration vectors at every instant.

Know what types of movement correspond to each graph.

Know how to solve problems related to ascending and descending movements, as well as when and where two moving objects meet.

Then you can answer the following test: Evaluation

## Movement (II) <br> Multiple choice test

1 Choose the correct statement about the intrinsic components of acceleration.
Tangential acceleration indicates the variation in the direction of the velocity vector.
Normal acceleration indicates changes in the module of the velocity vector.
Total acceleration is equal to the tangential acceleration plus the normal acceleration.
$\square \left\lvert\, \begin{aligned} & \text { It makes no sense to speak of them in movements that are not } \\ & \text { curvilinear. }\end{aligned}\right.$

2 The area under the $v / t$ curve


When an object is thrown upwards
प the velocity vector is always oriented the same way.
the acceleration vector changes constantly.
the acceleration vector remains constant.

4
In the visual 4.3, each time that the car and the motorbike meet on the screen
the meeting point corresponds to a meeting point "in reality".
they have covered the same space.
the meeting point does not necessarily correspond to a "real" meeting point.
they are moving at the same speed.

The point of intersection of the s/t graphs of two objects moving in the same direction, indicates

the spot where the objects meet.
the spot where the speeds are equal to each other.
$\square$
the spot where both objects have covered the same space.

The s/t graphs of uniformly accelerated circular movements

|  | are straight |
| :--- | :--- |
| $\square$ | are circular. |
| $\square$ | are curved |

The smaller the time interval

the more the average acceleration vector approaches the average velocity vector.
the greater the average acceleration vector.
the more the average acceleration approaches zero.
the more the average acceleration vectors value tends towards a definite value for each instant.

8
An object is moving with a 20 metre radius UCM. If it takes the body 10 seconds to complete a revolution, then its linear speed is

$0,1^{*} \mathrm{pi} \mathrm{m} / \mathrm{s}$.
$0.2^{*} \mathrm{pi} \mathrm{m} / \mathrm{s}$.
4*pi m/s.
$0,4^{*}$ pi m/s
is straight if the trajectory is straight.
is curved if the trajectory is curved.
$\square$
is always a parabola.
$\square$ is usually curved.

The a/t graph of a UARM


प is curved if the trajectory is curved.
प is always parallel to the t axis.
$\square$ is always a slanting straight line.

An object is moving with a 20 metre radius UCM. If it takes the body 10 seconds to complete a revolution, then its speed is approximately

|  | $45 \mathrm{~km} / \mathrm{h}$. |
| :--- | :--- | :--- |
| $\square$ | $4,5 \mathrm{~km} / \mathrm{h}$ |
| $\square$ | $0,4 \mathrm{~km} / \mathrm{h}$ |
| $\square$ | $0,04 \mathrm{~km} / \mathrm{h}$. |

## 12

An object is moving with a 20 metre radius UCM. If it takes the body 10 seconds to complete a revolution, then its angular velocity is

|  | $0.2^{*} \mathrm{pi} \mathrm{m} / \mathrm{s}$ |
| :--- | :--- | :--- |
| $\square$ | $0,4^{*} \mathrm{pi} \mathrm{m} / \mathrm{s}$ |
| $\square$ | $0,1^{*} \mathrm{pi} \mathrm{m} / \mathrm{s}$. |
| $\square$ | $4 * \mathrm{pi} \mathrm{m} / \mathrm{s}$. |

13 The v/t graph of a UARM:
$\qquad$ is always a slanting straight line.
is curved if the trajectory is curved.
$\square$
is straight if the trajectory is straight.
is always a parabola.

14 The average acceleration vector between two points is

|  | parallel to the displacement vector. |
| :--- | :--- |
| $\square$ | perpendicular to the position vector. |
| $\square$ | perpendicular to the average velocity vector. |
| $\square$ | parallel to the average position vector. |

15 The length of arc covered by a moving object with a circular movement
प remains constant in circular movements.
$\square$
is in inverse proportion to the radians covered.
$\square$ is in direct proportion to the radians covered.


If a body is freely falling vertically, then

its acceleration is constant.
$\square$
its acceleration is zero.
$\square$
its average acceleration is variable.
$\square$ its average acceleration is greater the smaller the time interval.
18 The s/t graphs of Uniform Circular Movements

| $\square$ | are circular. |
| :--- | :--- | :--- |
| $\square$ | are curved. |
| $\square$ | are straight. |

19 An object is moving with a 2 metre radius UCM. It covers an angular distance of 3 radians. The linear distance covered is therefore

| 0,66*pi m. |
| :---: |
| 1,5 m. |
| 6 m . |
| 12*pi m. |

20
The angular velocity of a body

is expressed in r.p.m.
is expressed in radians per second.

is expressed in Hz .
$\square$ is expressed in r.p.s.

