GRAVITATIONAL FIELDS

Presentation of the unit



According to the legend, the fall of an apple inspired Newton to create the theory of gravitation. No such thing ever happened.

The toilsome development of modern physics in the seventeenth century, which was often obstructed by prejudice and by different interests, finally culminated after a long process in the theory of gravitation.

It was the first body of physical theory developed according to our present conception. This is why the study of this subject is especially interesting.

Furthermore, the law of universal gravitation has allowed the human race to begin to understand the cosmos in which we live.

Objectives

- To understand the process that led to the establishment of the law of universal gravitation and its relation with Kepler's empirical laws.
- To explain the importance of the law of gravitation in the study of the movement of astronomical objects in the solar system, artificial satellites and tides.
- To learn the concepts of field intensity and gravitational potential energy.
- To understand how phenomena like Earth's rotation may change our perception of gravity.

 To be able, in simple cases, to carry out predictions about the values of field intensity, orbital data for satellites and energy in gravitational phenomena.

Freely falling bodies

Drop the apple in the visual. **Galileo** studied this type of movement, which was the origin of Newton's idea that Earth attracts all other bodies towards itself.



Galileo was even able to take air resistance into account. Click on <u>Next</u> to follow in his footsteps.

Click on Help for an explanation of the visual.



Help: This visual represents the movement of freely falling bodies from a height of 4 m above the ground. You may alter the shape and mass of the body and the density of air. Activities A1, A2 and A3 will help you to make the most of the visual.

A1: With a normal density, drop differently shaped bodies and see which arrives first. Try to make up a hypothesis that explains this fact.

A2: Choose the shape and mass of the falling body. Gradually decrease the density of air and note down the time taken to reach the floor in each case. When is the time minimum? Now change the mass and shape of the body and repeat the experiment. What do all the experiments have in common?

A3: What can you tell about the type of movement from the shape of the graph? Calculate the acceleration taking into account that the initial height is 4 metres. How must the body be shaped if its fall in air is to be similar to its fall in a vacuum?

Two ways of understanding the universe



Nowadays, we are familiar with the idea of a sun surrounded by a cohort of planets, asteroids and comets which orbit around it. Obtaining this knowledge, however, was not an easy task.

For a long time there were **two opposing theories** regarding our part of the universe: The **geocentric theory** and the **heliocentric theory**.

The fight between these two systems was harsh and tinted with ideological interests. In the following visual, you can study a version of each of them just as they were conceived at the dawn of the seventeenth century.



A1: Select the Copernican model and watch the simulation. What do the planets move around? What planets move faster?

A2: Select Tycho Brahe's model. When we choose this model, we can see the solar system as it is seen from our planet (as we feel that we are at rest). What can you see regarding the sun's movement? What about the planets' movement?

A3: Closely observe the movement of Mars in Copernicus' model and then in Tycho Brahe's model. Can you see any difference in the speed of the planet? Can you explain the reason for this difference?

Kepler's law



After the victory of the heliocentric theory, a new prejudice had to be defeated.

Both **Copernicus** and **Galileo** assumed that the orbits of the planets had to be circular, because circles are "perfect" shapes.

Kepler was able to defeat this prejudice of "circular perfection" by using the data he obtained from his own observations and those of his master, **Tycho Brahe**, the last defender of the geocentric theory.



Help: This visual represents the orbits of the planets Mercury, Venus and Mars. The initial distances and periods are actually correct, although the elliptical character of the orbits has been exaggerated to make the comprehension of Kepler's laws easier. You may choose the law that the visual will illustrate by choosing the number of the law from the drop down menu. You may also see a representation in colour of the area swept by the planet every 540 hours (22.5 days). The controls A1, A2 and A3 will help you explore the visual.

A1: Choose law number one and click on play. Watch the trajectory of the planet and note down the data that the visual shows at the end of the first revolution. Click on init and repeat the observations for the other planets. What is the meaning of each datum shown?

A2: Choose law number two and set the area control to show. When you click on play, you will see the area swept by the position vector in equal intervals of time. Note that the speed of the planet compensates for the differences in the distance to the sun at different points of the orbit. What is the meaning of the areolar velocity shown at the end of the revolution?

A3: Choose law number three. Click on play and note down the data at the end of the revolution. Click on play and repeat the observation with the other planets. What are the similarities?

Conclusions about the historical background of the law of gravitation

The low of freely felling	All bodies fall to earth with the same acceleration
hedies	when the resistance of air is negligible. This
boules	acceleration is g = 9,8 m/s ²

Kepler's Iaws	1 st The planets orbit the sun following flat elliptical paths, with the sun at one focus.
	2 nd The position vectors sweep equal areas in equal times.
	3 rd The square of the time it takes to complete a revolution is
	proportional to the cube of the major axes of the orbits.

The law of universal gravitation



The accelerated fall of bodies to earth led **Newton** to assume that our planet attracted all the bodies that surround it towards its centre.

Kepler's laws of planetary motion convinced Newton that the sun in turn attracted all the planets towards itself.

Newton's genius lies in his ability to reach a general rule: **the law of universal** gravitation.

Newton's law of universal gravitation and its first consequences



The law under the image on the left shows the force of attraction (**F**) between two astronomical objects of masses **M** and **m**, which are separated by the distance

$$\mathbf{F} = \mathbf{G} \cdot \frac{\mathbf{M} \cdot \mathbf{m}}{\mathbf{r}^2}$$

r.

The gravitational constant **G** was measured years later by **Henry Cavendish**. Click on **Measuring G** to see how it was done.

Measuring G



Help: Large masses: 158 kg; small masses: 0.75 kg. Length of the bar: 1.8 m. Torsion coefficient: 0.0003 N-m. Right-click on each control to see how it is used. Warning: the movements in the visual have been exagerated to better illustrate the experiment.

A1: Cast a ray of light and release the system. Explain why the weights move and why the ray of light changes its trajectory. You can move the system around to observe it better. Change D1 and D2. Note down the changes in the results. Can you explain these changes?

A2: Use your textbook to find out how this experiment was used to measure G. Click on

help to get the data needed to calculate the torque, the force and G. Check your results with the show drop down menu.

A3: Take one of the values of the deviation of the ray of light. Note that Cavendish could not measure this deviation with a precision greater than 0.2 mm. Use the greatest and smallest possible deviation to calculate G taking into account this margin of error. What is the margin of error in G? How can we make this margin of error decrease? An interesting point to remember is that, as the force of attraction between the two is mutual, both bodies move around their common centre of mass. However, when one of the astronomical objects is much greater than the other, the distance between the centre of mass and the centre of the large object is small.

Click on **Mutual attraction** and change the masses of the star and planet to check this statement.

Mutual attraction



The principle of superposition



Newton's **law of universal gravitation** can be applied to two objects.

When we are dealing with systems with more than two objects, we must use the **principle of superposition**: *The*

force exerted by one object on another is independent of the forces exerted by the other objects.

In sets like the solar system, where most of the mass belongs to only one of the heavenly bodies, the forces between the planets are in many cases negligible.

Click on <u>Next</u> to study a case in which this force should not be neglected. The visual shows a small asteroid moving between the sun and a big planet (Jupiter, for example).



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Help: A planet similar to Jupiter is orbiting the sun. An asteroid with a negligible mass (represented by the red dot) is also orbiting the sun, but its orbit has a smaller radius. The centre of the visual is at the centre of mass of the system.

You can change the initial speed of the asteroid and the masses of the planet and the sun. You can also choose whether to show the trajectory of the asteroid with the show orbit control.

The program shows the distance to the sun (the unit is equal to the orbital distance of Jupiter) and the time that has gone by.

The clear button allows you to erase the trajectory of the asteroid. You should click on init before you change the values of the speed and masses, otherwise the results will not be reliable.

A1: Set the mass of the planet to zero. Allow the asteroid to follow different orbits (changing the initial speed).

Are there any values of the initial speed that make the asteroid crash into the planet?

A2: Set the mass of the planet to 10 or 12 units, with the initial speed of the asteroid set to 1. What is the effect of the perturbation caused by the planet?

A3: Give the mass of the planet bigger and bigger values and decrease the value of the mass of the sun. What can you say about the stability of the orbit of the asteroid? What happens to the position of the sun?

Angular momentum and central force



Newton's law of universal gravitation is designed to satisfy **Kepler's** third law.

The other two laws are related to the concepts of angular momentum and central force.

The angular momentum (L) of a particle

about an origin O is the vector product of the position vector r and the linear momentum $m \cdot v$.

The variation of L depends on the torque acting on the particle. If the force causing the torque is a central force, then the angular momentum of the particle about the centre is equal to zero and the angular momentum remains constant.



Help: This visual represents a particle under the influence of a central force. The red vector is the velocity, the green vector is the force exerted on the particle and the yellow vector is the angular momentum. You can change the mass and the components of the velocity vector. You can also change the value of the central force pointing towards O.

A1: Set the value of the central force to zero and click on play. How does the particle move? Are there any changes in the value of the velocity vector or the angular momentum vector?

A2: Set the force to 5 N and click on play. How does the particle move? What variations can you see in the velocity, force and angular momentum vectors?

A3: Change the components of the velocity vector in such a way that it is initially pointing upwards. What happens to the angular momentum? (drag to see the vectors properly) What happens when you increase the mass? Set the force to 5 N and click on play. How does the increase in mass affect the movement?

A4: Compare the movement of the particle to the movement of the planets around the sun. Can you see the similarities? In the case of planetary motion, what is the central force?

Conclusions about the law of universal gravitation



The principle of	When we deal with systems with more than two bodies,
	the force exerted by one body on another is
superposition	independent of the forces exerted by the others.

Angular momentum and central forces	The angular momentum L of a particle about an origin is
	the vector product of the position vector and the linear
	momentum. If the only forces acting on the particle are
	central, the angular momentum remains constant.

The concept of field intensity

ģΡ

М

 $\vec{g} = G \cdot \frac{M}{R^2} \cdot \vec{u}$

Modern physics considers the gravitational field an alteration of the properties of the space surrounding bodies.

This alteration is measured by the **field intensity**, which is defined as *the force felt by a unit of mass at a point in the field*. The intensity of the gravitational field near the surface of the Earth is a vector of approximately 9.8 N/kg towards

the centre of the Earth.

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The vector **g** in the figure above measures the field intensity of gravity created by a body of mass **M** at a point P outside the sphere at a distance **R** from the centre. The unit vector **u** points in the direction of the intensity.

Click on <u>next</u> to see a graphic representation of the field created by a body and the effect of the principle of superposition when there is more than one body producing the field.



Help: Type in the masses of one or two bodies (m1 and m2) and press enter. The unit will be the mass of the Earth (Mass of the Earth = 1). The visual will draw arrows that show the direction of the field outside the bodies. You can drag the red dot to different points in the field to see the field intensity vector and its value in N/Kg.

A1: Set the value of m1 to something greater than zero. You will immediately see the graphic representation of the field. Observe the direction of the field intensity vectors. Would you be able to draw the lines of force?

A2: Give m1 and m2 similar values (greater than zero) and observe the direction of the field intensity vectors. What would the lines of force look like now? Repeat the experiment giving m1 and m2 very different values.

A3: Give m1 and m2 values greater than zero and move the red dot around the visual. What happens to the value of the field intensity? Can you guess where there is a point in the field where the field intensity is zero?

Variations according to position

The famous expression for the field intensity of gravity is only valid if we are calculating the field intensity outside the planet. Inside the planet, it is possible to prove, using **Gauss'** theorem, that the **field intensity depends only on the mass that is nearer to the centre of the planet** than the point at which we are measuring the field.

Click on **planetary g** to study the field intensity of gravity inside and outside a planet.



Help: This visual graphs the field intensity of gravity inside and outside the planet.

The initial values are equal to those of the Earth. Drag the white dot around the visual, you will see the value of the field intensity at the point marked by the dot. On the right you will see a graph of the different values of the field intensity for different values of the radius. You should click on init each time you wish to change the values of the mass and the radius of the planet.

A1: Without changing the position of the observation point (on the surface of the planet) or the radius of the planet, note down the values of the field intensity (g) for different values of the mass. What is the relation between these two magnitudes?

A2: Keep the observation point on the surface of the planet and decrease the value of the radius. What happens to the field intensity ? What if we give the radius values

greater than one? Try to come up with a hypothesis that explains these results.

A3: Keep the observation point on the surface of the planet and decrease the value of the radius. What happens to the field intensity ? What if we give the radius values greater than one? Try to come up with a hypothesis that explains these results.



Furthermore, on the surface of a rotating planet, it is necessary to take into account the apparent **change in the force of gravity due to the centrifugal force** caused by rotation.

Click on apparent g to observe these effects.



Help: This visual represents the influence of the Earth's rotation on the value of the field intensity measured. You can change the latitude of

the observation point and the speed of rotation of the Earth. Imagine the plumb line on the right suspended over the observation point on the planet.

A1: Change the latitude of the observation point and observe the apparent change in the field intensity and the change in the angle of the plumb line. Where is there no deviation of the plumb line? Try to explain these observations with the help of the concept of centrifugal force.

A2: Set the latitude to zero. Decrease the period of rotation. How does this influence the observed gravity? Repeat the observation for a temperate latitude (40°) and at the pole (90°). Try to explain your observations.

Conclusions about the field intensity



	proportion to its distance to the centre of the planet.
planet	value of the field intensity inside the planet increases in direct
inside a	than the point at which we are measuring the field intensity. The
Gravity	on the part of the mass of the planet that is nearer to the centre
	The field intensity of gravity at a point inside a planet depends only

Gravity on a	The rotation of a planet causes an apparent variation in the
rotating	force of gravity measured on the surface of the planet, due
planet	to the action of the centrifugal force.

Gravitational potential energy

The take off of a space shuttle uses up a great amount of energy which is stored in the form of **kinetic and potential energy**. We already know the value of the potential energy for relatively low heights: Ep=m·g·h, which is a measure of the work carried out by the gravitational field on a body dropped from a height of h metres.



When we consider greater heights, it is no longer possible to assume that the acceleration of gravity is a constant and the problem becomes more complicated.

Click on **potential energy** to see how to calculate the potential energy. The activities in the visual also define the **gravitational**

potential, a magnitude which is characteristic of the field, as is the field intensity.

potential energy



Help: This visual calculates the work carried out by the force of gravity on a shuttle that is flying away from the Earth, dividing the distance into as many intervals as you wish. The force of gravity is considered constant within each interval. It then compares this value to the value you would get if you used the formula Ep=-G·M1·M2/r, where M1 and M2 are the masses of the planet and shuttle. The mass of the planet is measured in Earth masses for conciseness. You can choose the starting and end points of the journey by dragging the red dots to different points.

A1: Without changing the number of intervals, click on play. Note the difference between the value calculated multiplying the force by the distance and the value calculated at the bottom of the visual. Repeat the experiment increasing the number of intervals to 50, 100, ... Why do the values of the work calculated by different means approximate each other?

A2: You have probably realized that the work has a negative sign. Click on init and move the red dots in such a way that the rocket moves back towards Earth. What happens to the sign? Why?

A3: Use the visual to calculate the work carried out by the force of gravity when the shuttle moves from 10.000 to 20.000 Km and from 20.000 to 30.000 km. Do you get the same result for the work? Why?

A4: The potential energy of a unit of mass at a point is called the gravitational potential at that point. If you make M2 = 1 kg, the values of energy measured are those of the gravitational potential. What magnitudes does the gravitational potential depend on?

Equipotential surfaces and lines of force



The surfaces formed by points with the same gravitational potential are called **equipotential surfaces**.

Lines of force are lines which are tangent to the field intensity vector at every point in the field.

Equipotential surfaces and lines of force are perpendicular to each

other.

n the visual above, you can see the equipotential surfaces and lines of force in the field created by the planet.

Help: This visual shows the potential and field intensity at any point in the visual.

You can make the visual draw equipotential surfaces (represented by a curve) and/or lines of force.

You can also change the value of the mass.

Gravitational potential

A1: Drag the red dot around the visual and observe the variation of the intensity and potential inside and outside the planet. Can you see any differences in the variations? What causes them?

A2: Choose the lines of force option from the drop down menu.

A3: Choose the equipotential surfaces option from the drop down menu.

	The potential energy of a body placed at some point in the field measures the work carried out by the field if that body were to move away to infinity.
Gravitational potential energy	For a body of mass m , at a distance r from another body of mass M , the potential energy is equal to $E^{n}p = -G \cdot \frac{M \cdot m}{r}$ If there are several bodies to consider, the principle of superposition must be used.

Conclusions about gravitational potential energy

<u>I</u>	
Equipotential	The equipotential surfaces are formed by points with
surfaces and lines	the same value of potential and they are

It is the potential energy per unit of mass.

of force	perpendicular to the lines of force, which show the
	direction of the field intensity.

Explanation of the tides

The **tidal force** exerted by one body on another is a direct consequence of gravitation.

Actually, the tidal force is the difference between the field intensity of gravity created by a body at the centre of another, and the field intensity it creates on the surface of this other body.



The Earth suffers detectable tidal forces produced by the sun and moon. The combination of both forces, together with the rotation of the Earth and the movement of the Earth and moon, create the variations in the tide that sailors know so well.

Click on Next to observe and understand these peculiar phenomena.



A1: Change the day of the lunar month and observe the corresponding tide. How many zones of high tide appear on the earth? Where are they situated?

A2: Press the animation button. The tides do not always rise by the same amount. In which cases is the high tide higher (high coefficient)? When are there dead tides (low coefficient)? Invent a hypothesis that explains it.

A3: Activate the observer and change the solar time. How many high tides does he see during the day? Press the animation button. Do the high tides always come at the same time of the day? How do tides move along the surface of the earth?

The movement of artificial satellites



Artificial satellites, and their uses in research and the media, are possible thanks to our knowledge of the laws of gravitation.

The force of gravity keeps the satellites in a previously

calculated orbit. Study their movement by clicking on satellite.



Help: A satellite, represented by a blue dot, moves in orbit around the Earth when you click on play.

You can change the initial height and speed when the engines stop. The Sp control sets the speed of the simulation. A fast simulation is less precise.

You can make the orbit of

the satellite visible with the V. o.? control, and you can erase it with the clear button.

A1: Set the initial height to the minimum allowed by the visual (500 Km). Test different initial speeds until you get an orbit which is approximately circular. Note down the initial speed for this orbit and the time it takes the satellite to complete a revolution. If you increase the speed, what happens to the orbits? Try to find the minimum speed at which it escapes the gravitational field of the Earth. This speed is known as the escape velocity. What is the mathematical relation between the two speeds?

Repeat the experiment for initial heights of 1000 and 5000 km. Can you derive a general conclusion?

A2: You may use the data from the previous activity or take new measurements. How does the period change with the height? At the right height, a satellite would take exactly 24 hours to complete a revolution around the Earth. If the satellite were over the equator, how would its movement be seen from the Earth?

A3: Set the value of the initial velocity in such a way that the orbit is an longated ellipse. What happens to the speed along the orbit? Try to note down the speed when the satellite is at the most distant point from the Earth and compare it to the value when it is closest to the Earth. Multiply each of these values by the distance. What can you observe?

You can also study the conditions necessary for a satellite to escape the



attraction of the Earth by clicking on

Help: A shuttle is launched vertically from the surface of the Earth. We will assume that the fuel runs out at a very low height. The blue curve represents the potential energy of the shuttle as a function of the distance r from the centre of the planet.

On the graph, the total energy of the shuttle is plotted against its distance from the centre of the Earth. After the shuttle is launched, its speed decreases because of gravitational attraction.

A1: Click on play. When does the shuttle stop?

A2: Launch the shuttle again and gradually increase the speed. At what speed does the shuttle escape? What happens to the total energy then? What physical significance can we attribute to the potential energy?

One last question:

You have seen that gravity is responsible for keeping satellites in their orbits around the Earth. Why do bodies float inside satellites, then? Remember that they must be under the influence of the gravitational field of the planet. Discuss the question with your classmates, and try to find the answer.



Coherence of galaxies



This is a photograph of the galaxy M100, a **spiral galaxy like ours.** Other galaxies are **spherical or elliptical.**

All galaxies have in common the fact that it is the force of gravity that keeps the thousands of millions of suns they are made up of travelling around their common centre of mass.

Why are there galaxies with different shapes? We now know that collisions between galaxies modify their shape.

However, their initial conditions are also very important in the formation of their final shape. Click on **galaxy** to see the original cause of these differences.



Help: This visual represents a galaxy with only 200 stars.

They all rotate around the centre of mass of the system shaped like a balloon. An increase of the angular momentum means that at least some of the stars will travel faster, which means they will be at a greater distance from the centre.

Conclusions about the force of gravity and the coherence of galaxies

	A galaxy is made up of thousands of millions of stars in gravitational interaction.
The role of the force of	
gravity	The force of gravity among its stars is what keeps
	the galaxy together, with all the stars travelling
	around the centre of mass.

	The total angular momentum of the galaxy about its
	centre of mass is what determines its shape. Galaxies with
The role of	a small angular momentum tend to be spherical, while
angular	galaxies with a greater angular momentum tend to be shaped
momentum	like a disc.
	The shape of a galaxy can be modified due to its interaction with other galaxies.

Evaluation

- Do you understand the process which led to the establishment of the law of universal gravitation and its relation to Kepler's empirical laws?
- Can you explain the importance of the law of gravitation for understanding the movements of the astronomical objects in the solar system, of artificial satellites and of the tides?
- Do you understand the meaning of the concepts of field intensity and potential gravitational energy?
- Do you understand how local phenomena such as the rotation of the Earth can alter our perception of gravity?
- Can you make predictions about the values of field intensity, the orbital data of satellites or energy, in simple cases of gravitational phenomena?

You can check your knowledge by answering the following questions:

Evaluation

Choose the correct answer to each question

- 1 The field intensity of gravity
 - decreases when we move away from the surface, and it is constant inside the planet.
 - is always equal to 9,8 N/kg
 - is minimum on the surface of the planet.
 - is maximum on the surface of the planet.

- 2 The original shape of galaxies, before they interact with other galaxies, depends on
 - their total angular momentum about the centre of mass of the galaxy.
 - the number of stars (spiral galaxies are always the largest).
 - the average force of gravity among the stars that form the galaxy.
 - the average mass of the stars in the galaxy.
- 3 The rotation of the planets
 - does not modify the gravitational field in a way that can be perceived.
 - does not change the value of the field measured by machines.
 - modifies the apparent value of the gravitational field on the surface of the planet (except at the poles).
 - modifies the apparent value of the gravitational field on the surface of the planet (except at the equator).
- 4 The potential energy of a body in a gravitational field
 - decreases as its distance to the planet increases.
 - is always positive.
 - increases as its distance to the planet increases.
 - is independent of the distance to the planet.

5 When a large planet is near a smaller one

the force with which it attracts the smaller planet is equal to
the force with which the latter attracts the former.

the force with which it attracts the smaller planet is greater than the force with which the smaller planet attracts the large one.

the force with which the smaller one attracts the large one
is greater than the force exerted by the large one on the
smaller one.

the force exerted by each of them depends on the medium.

- 6 What condition must the speed of a body satisfy to escape the attraction of a planet?
 - Its speed must be equal to or greater than 11,3 km/s
 - Its potential energy plus its kinetic energy must be equal to or greater than zero.
 - Its kinetic energy plus its potential energy must be equal to zero or a negative number.
 - Its speed must be equal to the square root of G·M/R where M is the mass of the planet and R is the radius.

7 The law of universal gravitation is constructed so that

- The force is inversely proportional to the distance between the bodies.
- The force between two bodies is independent of the distance between them.
- Kepler's empirical laws are satisfied.
 - The force is proportional to the distance between the bodies

- 8 Does angular momentum play a role in the movement of the planets around the sun?
 - Yes, the conservation of angular momentum is responsible for Kepler's third law.
 - No, planetary movement is related to the force of gravity and not to angular momentum.
 - Yes, the conservation of angular momentum is responsible for Kepler's first and second laws.
 - No, angular momentum is only relevant when we study the movement of objects that escape planetary attraction.

9 The speed of a satellite in orbit around a planet

- is such that the satellite always moves in circles around the planet.
- is proportional to the square root of the mass of the satellite and inversely proportional to the radius of the planet.
 - is approximately equal to 8 km/s.
 - is proportional to the square root of the mass of the planet and inversely proportional to the radius of the planet.
- 10 Can a satellite always be seen at the some fixed point in the sky relative to the surface of the Earth?
 - It depends on the observer's latitude.
 - No, because it would escape from the Earth.
 - Yes, if the orbital period of the satellite is the same as the orbital period of the Earth.
 - No, because it would fall to the Earth.

Try to apply the laws of gravity

- 1 A planet has a radius of 1000 Km. The field intensity of gravity is equal to 2 N/Kg on the surface. Calculate the escape velocity on the surface of the planet in Km/s.
- A satellite is at a distance R from the centre of the planet.
 The mass of the planet is 9 times the mass of the satellite. At what distance from the centre of the planet is the force of gravity nil?
- 3 The orbital periods of two satellites are 1 and 8 years respectively. What is the ratio of the major axis of the larger orbit to the major axis of the smaller one?
- 4 A comet moves at speed V when it is at the aphelion of its orbit, at a distance R from the sun. When the comet is at the perihelion at a distance R/5, its speed is
- 5 The potential energy of a satellite in a circular orbit around the Earth is equal to -E. How much energy must be added to make the satellite escape from the planet?
- 6 Given a planet of radius R, calculate at what distance above the surface the force of gravity is a quarter of the force on the surface (give your answer as a function of R).
- 7 Two satellites are orbiting the Earth. Satellite A moves at twice the speed of satellite B. Calculate the following ratio: orbital radius of satellite A/orbital radius of satellite B.

8 We take a 1 Kg object to another planet, which has a density which is twice the density of the Earth and with a radius equal to half the radius of the Earth. What is the weight (in N) of the object on the other planet?