UNIFORM CIRCULAR MOVEMENT

OBJECTIVES

Circular movement can be seen in many gadgets which surround us; engines, the hands of a clock and wheels are some examples. This teaching unit presents the characteristic magnitudes of Uniform Circular Movement and revises the concepts of arc and angle.

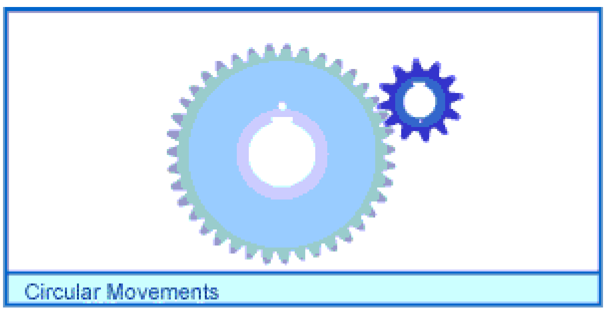
This unit concludes the study of movement which began in the previous three units:

Moving bodies Trajectory and Displacement Rectilinear Movements

To understand the kinematic characteristics of Uniform Circular Movement.

- To understand the meaning and utility of the radian for describing this movement.
- To express velocities in rad/s, r.p.s. and r.p.m. and to convert them from one to another.
- To understand the meaning of linear and angular magnitudes.
- To transform linear magnitudes into angular ones and viceversa.

1.1 Uniform Circular Movement: What is it?



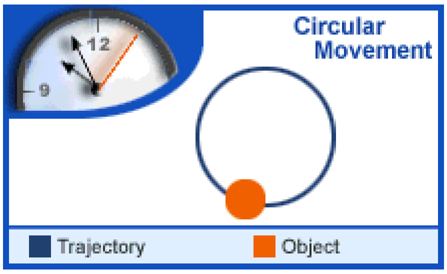
Gears, wheels,CD-ROMs, loops in big dippers, etc. etc. etc., circular movements surround us; we are only going to study the simplest of all these: uniform ones (those which move at a constant speed).

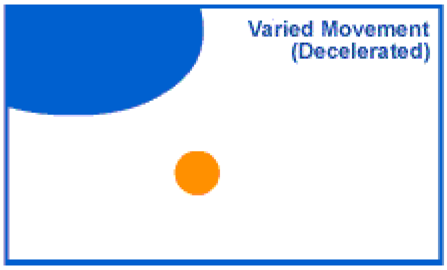
1.1.1 Uniform Circular Movement: What is it?

Different trajectories of circular movements.

A body moves round a circumference at a constant speed

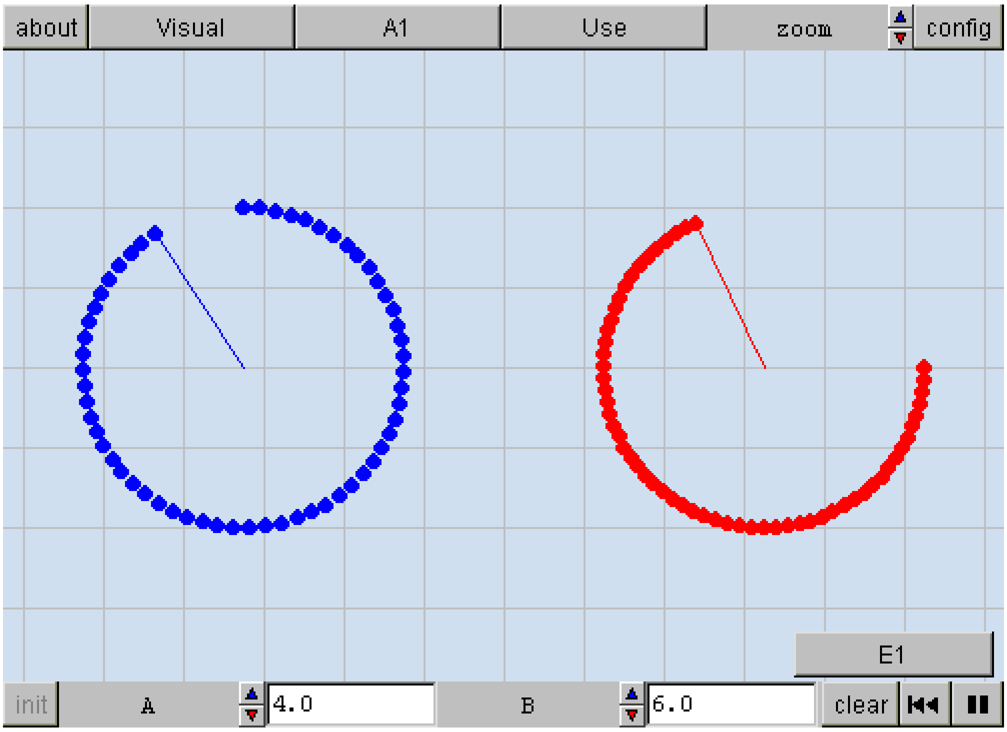
We cannot tell from the trajectory if the movement has always been at the same speed





1.1.2 Uniform Circular Movement: What is it?

Remember: its **trajectory** is a **circumference**, and also it goes at a **constant speed**.



VISUAL: Two uniform circular movements. You can modify the two movement controls.

A1: 1. Make them go at the same speed. 2. Make the red one go at half the speed of the blue one. (They are going at the same speed when they complete a revolution in the same time)

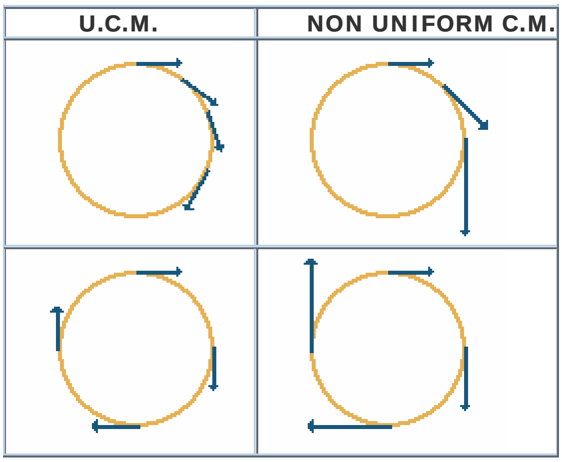
Use:

E1: Fill in the two missing words

1.2 Uniform Circular Movement: Does it show acceleration?

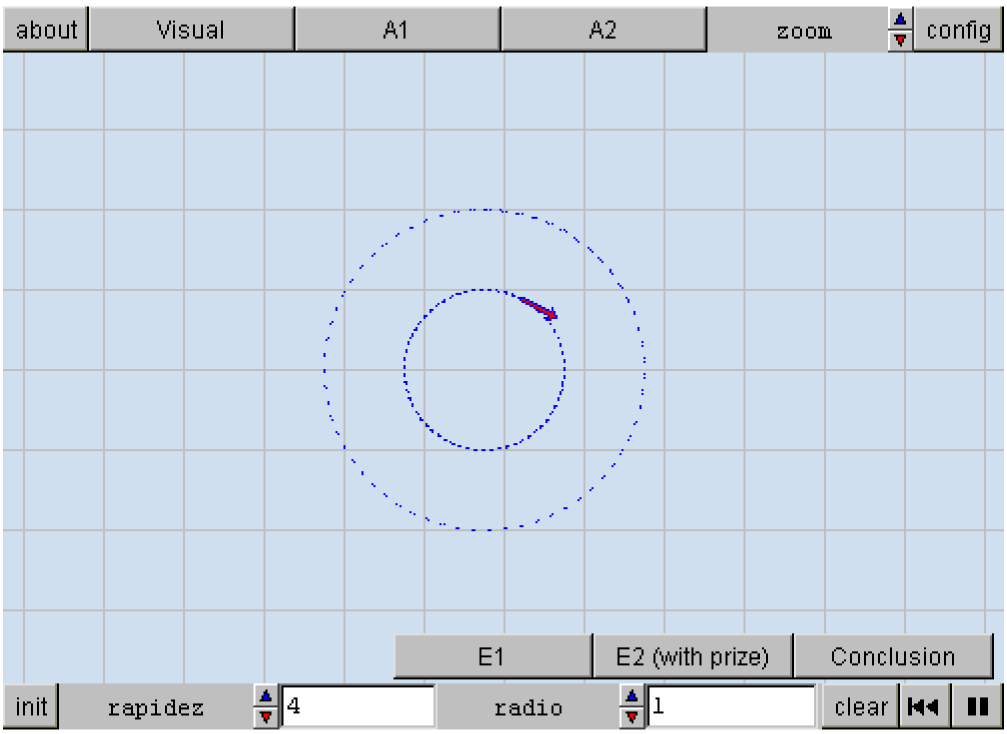
Although the circular movement is uniform and its ***speed*** is ***constant***, its ***velocity*** is ***variable*** and thus is accelerated.

Remember that ***speed*** is a scalar magnitude which ***does not change*** during the UCM, while ***velocity*** is a vector which ***does change*** constantly.



1.2.1 Uniform Circular Movement: does it show acceleration?

Speed does not change during the UCM, while ***velocity does change*** constantly.



VISUAL: ...in which you can vary the speed and the radius.

A1: ...make the speed minimum and the radius maximum.

A2: ... its velocity is perpendicular to the horizontal axis and moving upwards.

E1: What must we do to make the velocity vector opposite to the one in A.3?

E2: ... when the velocity is perpendicular to the vertical axis clockwise and then anticlockwise. That is the only way to get the prize.

2.1 How do we describe it? : revolutions per minute (r.p.m.)

If we know how many circuits are made per second or per minute, we can get an idea of how fast it is going.
The word **"revolution"** is sometimes used as a synonym of **"circuit",** so it is usual to express the speed of a U.C.M. in: r.p.m. (revolutions per minute) or r.p.s.: (revolutions per second).

A simple way to say how fast a U.C.M. is going is to express how long it takes to make a complete circuit.

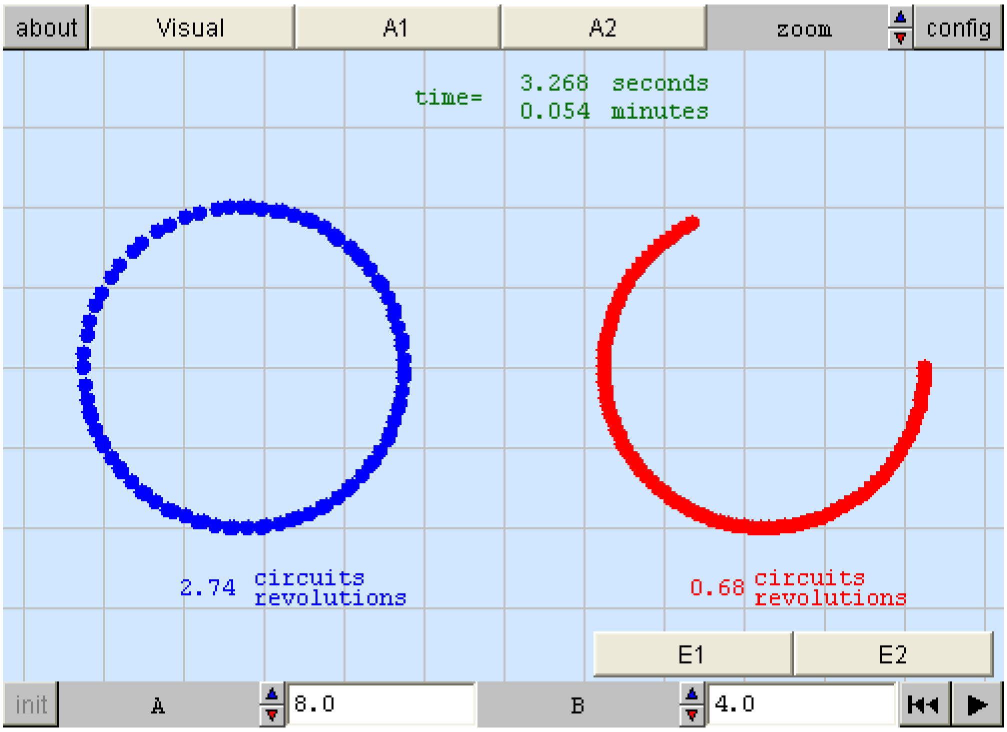
1. How long does it take the second hand of a clock to make a complete circuit?
Another way of expressing the speed of a U.C.M. is to say how many circuits it makes in one minute.

2. How many circuits does the second hand of a clock make in one minute? (r.p.m.)
You can also calculate the number of circuits made in one second.

3. How many circuits does the second hand of a clock make in one second? (r.p.s.)
You can repeat the exercise for the other two hands of the clock.

2.1.1 How do we describe it? : revolutions per minute (r.p.m.)

How fast are they going?



VISUAL: ...two uniform circular movements with identical radii. Modify the movement controls, especially the red one`s.

A1: ... until you make it go: 1) as fast as the blue one. 2) At half the blue one`s speed.

A2: Execute the computer

E1: Find out the r.p.m. and the r.p.s of A when it

E2: for B, when its control is set to 3.

2.2 How do we describe it? : radians per second (rad s^{-1})

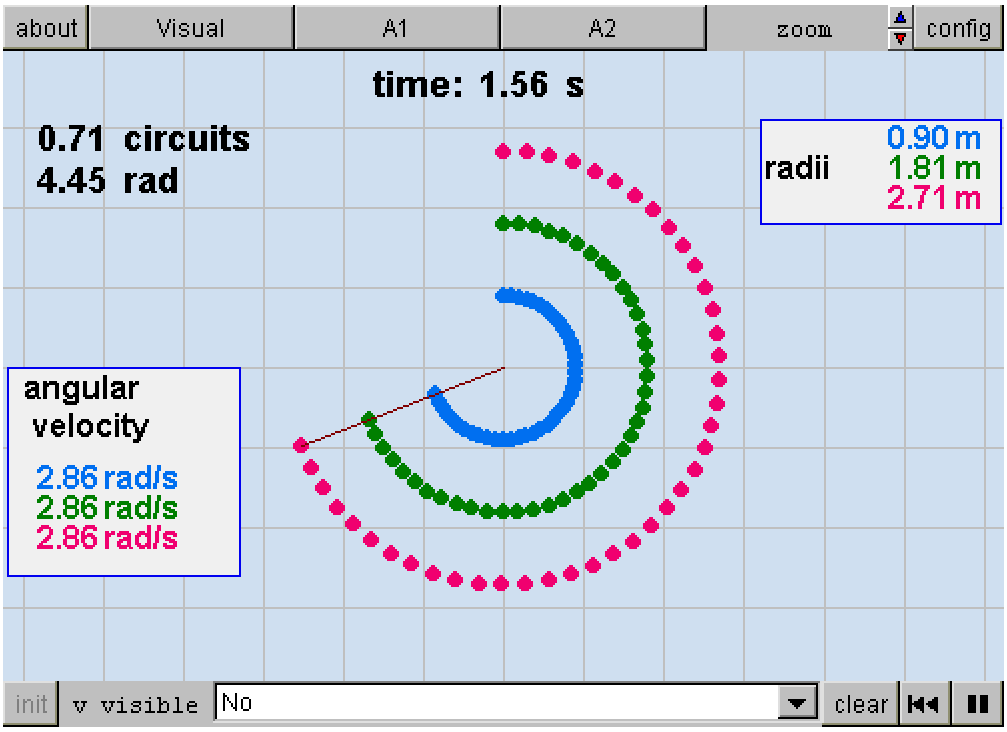
As well as r.p.m. and r.p.s., the U.C.M. can be described using the speed with which the angle described by the radius which joins the centre of the movement to the body, changes.

The way to express the units of speed of the UCM in the Système International d'Unités: that is, ***angular velocity***, is radians per second.

Of course, all the ways used to express speed are related. To understand this way of expressing angular velocity you have to know what a radian is .

To calculate angular velocity you just have to divide the **angle** described (ϕ ,in radians) by the **time** taken (t); $\omega=\phi/t$

2.2.1 How do we describe it? : radians per second (rad s^{-1})



VISUAL: Every point of the segment that moves has a linear velocity.

A1: has an angular velocity that is equal to the angular space (in radians) divided by the time.

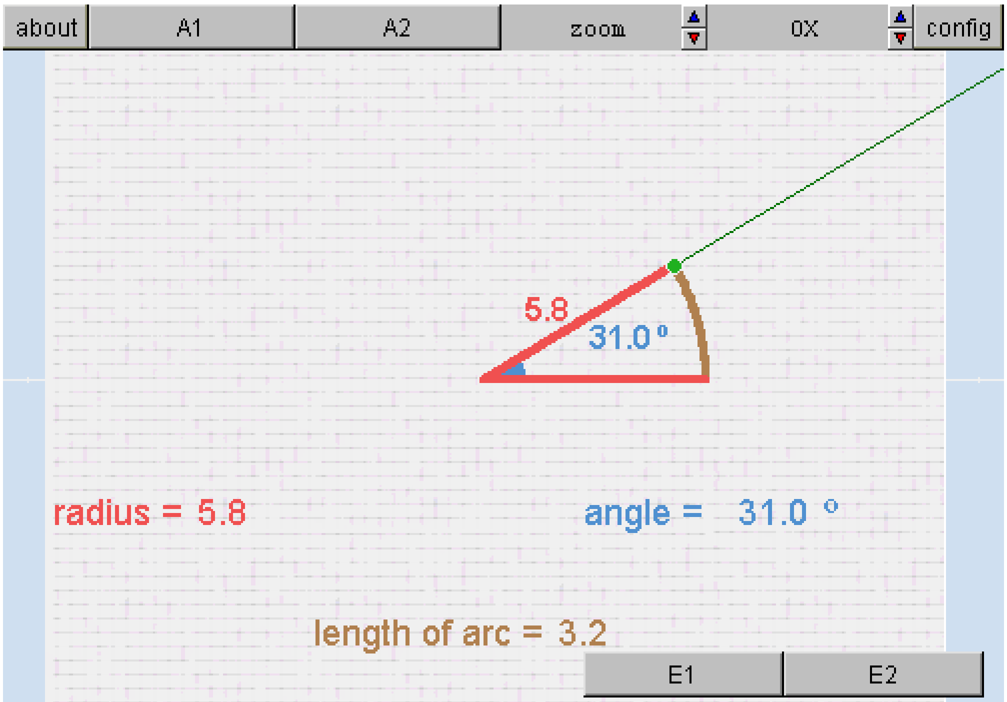
A2: has a linear velocity equal to the angular velocity times the radius.

3.1 What is a radian? : arc, angle and radius

You should revise the meaning of arc angle and radius if it is not clear to you.

angle: the space between two straight lines that diverge from a common point (vertex). A right angle has 90 sexagesimal degrees.

arc: is the circular line which surrounds the angle at the end of two segments.



A1: ... and drag it. Observe in what ways you can vary the angle, the arc and the radius at the same time.

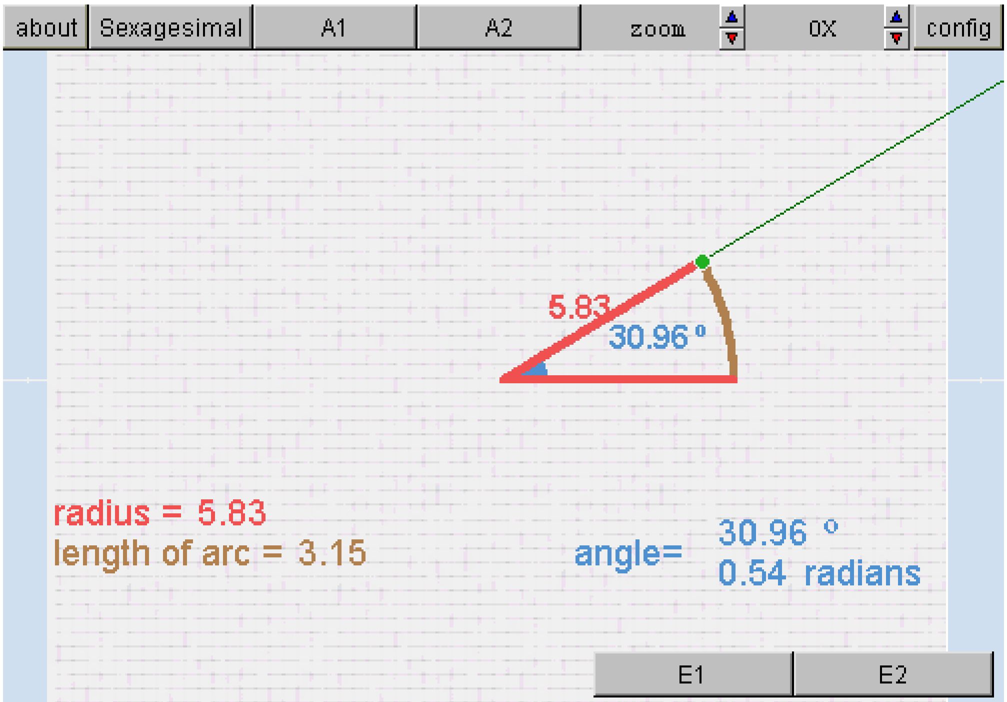
A2: ... make the angles equal to 90, 120, 180 and 270 sexagesimal degrees.

E1: ... be equal angles with different radii?

E2: different angles with the same length of arc?

3.2 What is a radian? How many radians are there in a circumference?

In Physics, angles are not usually measured using the sexagesimal system but in radians. The radian is the unit used according to the Système International d'Unités. The **radian is the angle whose arc has the same length as the radius.** The length of the arc corresponding to the whole circumference is 2*p*r How many radians will it have?



Sexagesimal: ... 360 sexagesimal degrees

A1: ... find out the value of different angles in radians

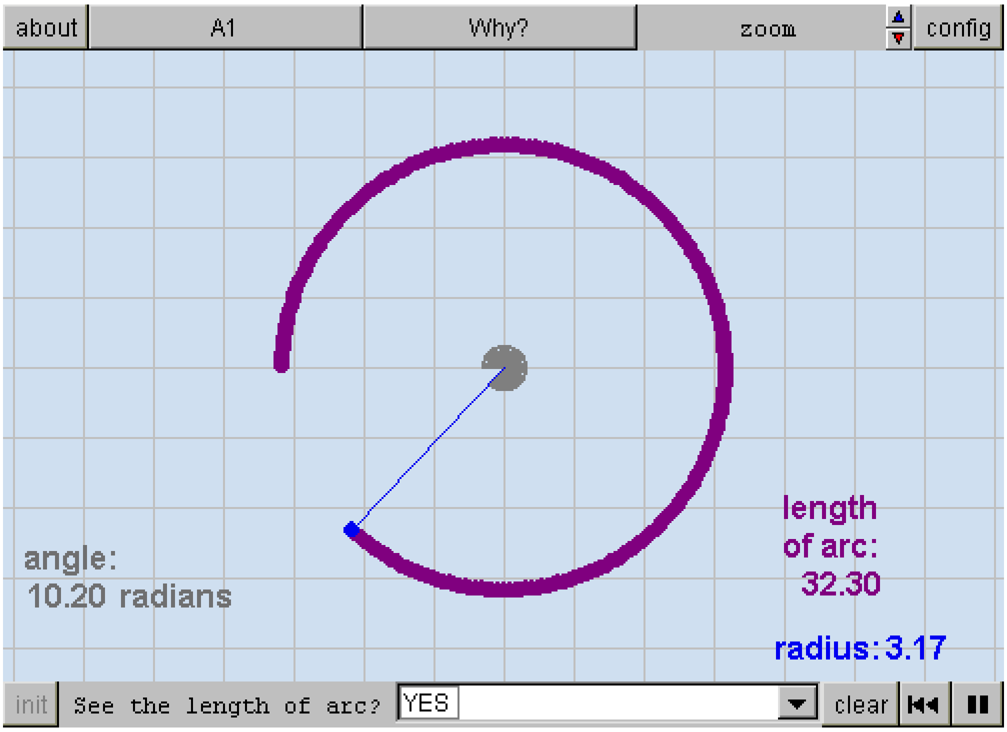
A2: ...that the measure of the angle in radians is the division of the length of arc by the radius.

E1: 90,180,270 and 360 sexagesimal degrees.

E2: ...the radians in an angle directly, we write pi/4, pi/2, (3/2)pi, 2pi. Calculate the sexagesimal degrees that correspond to each of the previous angles.

4.1 Linear and angular magnitudes. Linear space and angular space

A body displaying a circular movement covers a space (s) which can be measured in metres: **linear space, or distance covered,** and an angle ( ) which is measured in radians: **angular space**. These two ways of describing displacement are related; the radius of the movement is decisive in this relation. Observe that at all times the length of the arc s = * r



A1: ...verify that the length of arc is always the angle (in radians) times the radius.

Why?: If a circumference is 2pi radians long and it spans 2pi radians, then a radian is the arc whose arc has the length of the radius.

4.2 Linear and angular magnitudes. Linear velocity and angular velocity

We call the radians per second displayed by a body with UCM **angular velocity**, **w**. As well as describing an angle, the speed at which the arc is covered can be measured in m/s, this is **linear velocity**. The difference between these two ways of measuring velocity depends on the radius.

To calculate angular velocity you just divide the angle covered (□ in radians) by the time taken (t):

$$w = \phi / t$$

Given that $\phi = s/r$, when you substitute in the previous equation, you are left with	$\omega = \dfrac{\left(\frac{s}{r}\right)}{t}$

$s/r = v$	$\omega = \dfrac{v}{r}$	*or also*	$v = \omega . r$

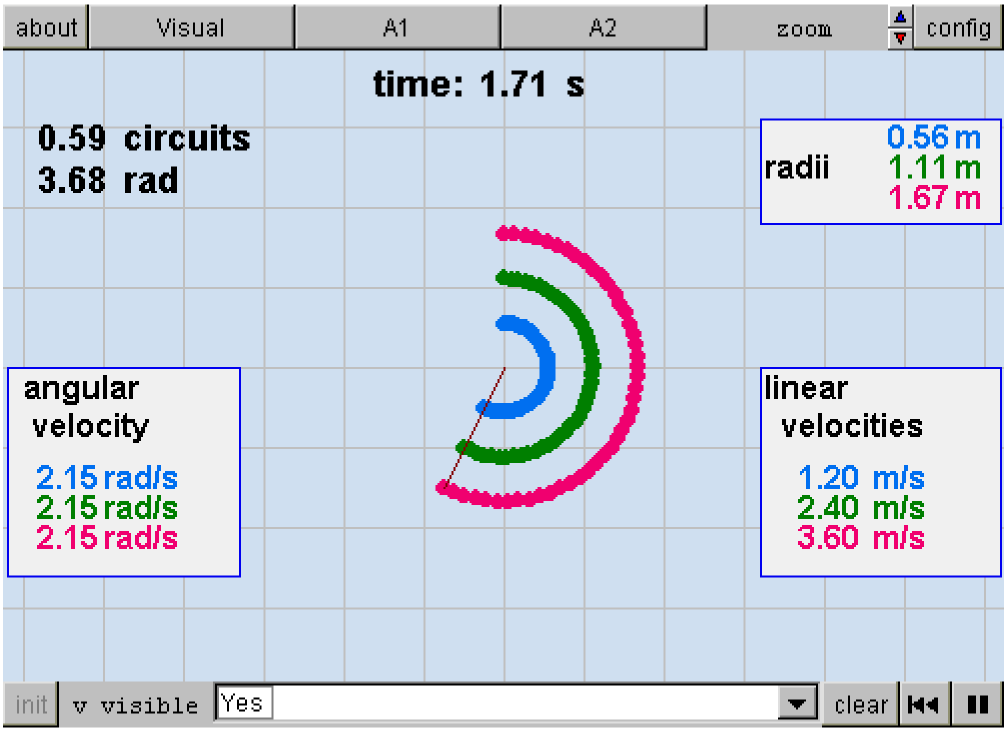
To summarise, in U.C.M.

	space	**velocity**
linear	$s = \phi . r$	$v = \omega . r$
angular	$\phi = s/r$	$\omega = v/r$

4.2 Angular and linear magnitudes. Linear velocity and angular velocity.

Calculate angular velocity.

$$w = \phi / t$$



VISUAL: Every point of the segment that moves has a linear velocity.

A1: ...has an angular velocity that is equal to the angular space (in radians) divided by the time.

A2: ...has a linear velocity equal to the angular velocity times the radius.

5. EVALUATION

Do you know the kinematic characteristics of Uniform Circular Movement?

- Do you know the meaning and the utility of the radian for describing this movement?
- Can you express velocity in rad/s, r.p.s. and r.p.m. and convert them from one into another?
- Do you know the meaning of linear and angular magnitudes?
- Can you convert linear magnitudes into angular ones and vice versa?

See if you can in Evaluation

Choose the right answer to every question

1 A body with U.C.M. completes one circuit every 60 s. Its angular velocity is:

- [ ] 1/60 r.p.s.
- [ ] 60 r.p.s.
- [ ] 2pi rad/s
- [ ] pi/30 rad/s

2 Uniform Circular Movements are accelerated...

- [ ] because the direction of the velocity vector changes.
- [ ] because the size of the velocity vector changes.
- [ ] only when it is not moving at a constant speed.
- [ ] because the size of the radius changes.

3 By looking at the visual **3.2**, we can infer that 64.44 sexagesimal degrees are equivalent to:

- [ ] 8.60 radians
- [ ] 1.12 radians
- [ ] 7.65 radians

4 A right angle measures:

- [ ] pi radians.
- [ ] 1 radian.
- [ ] pi/2 radians.
- [ ] 100 sexagesimal degrees.

5 A body is in a Uniform Circular Movement with a radius of 2 m. If it completes one circuit in one minute, its linear velocity in the Système International d`Unités is:

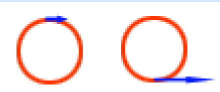
- [ ] pi m/s
- [ ] 1 m/s
- [ ] 4 pi m /s
- [ ] pi/15 m/s

6 Which of the following statements is FALSE?

- [ ] A radian is a unit that measures length of arc.
- [ ] A circumference covers 2pi radians.
- [ ] The radian is a unit that measures angles.
- [ ] Sexagesimal degrees can be transformed to radians

7 When you try to draw equal angles with different radii in the visual **3.1**:

- [ ] It's easy
- [ ] It can be easy or difficult, depending on where the vertex is.
- [ ] It turns out to be impossible

8  These images represent Uniform Circular Movements. The difference lies in:

- [ ] Their acceleration
- [ ] The direction of the movement.
- [ ] Their speed
- [ ] The radius of the movement

9 When the value of control A in the visual **2.1.1** is 4, the speed of the circular movement is:

- [ ] 2.37 r.p.m.
- [ ] 25.29 r.p.m.
- [ ] 0.039 r.p.m.

10 A body in U.C.M. completes 0.43 circuits in 0.034 minutes. Therefore, its speed is:

- [ ] 12.65 r.p.m.
- [ ] 0.464 vueltas por minuto
- [ ] 0.080 r.p.m.

11 A body in U.C.M. completes an angular distance of 1 radian in 15 seconds. Its angular speed is then:

- [ ] 15 rad/s
- [ ] pi/15 rad/s
- [ ] 1/15 rad/s
- [ ] 2pi/15 rad/s

12 In order to calculate the angle covered by a body in U.C.M., knowing the value of the radius, you only have to:

- [ ] equate the space covered to the radius
- [ ] multiply the space covered by the radius
- [ ] divide the space covered by the radius

13 If the visual is to be a good simulation of reality, the radii:

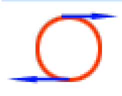
- [ ] can be expressed in any unit.
- [ ] must be expressed in centimetres.
- [ ] must be expressed in metres.

14 A body in U.C.M. moves at a angular velocity of 15 rad/s. After 15 seconds it will have moved:

- [ ] 225 radians
- [ ] 1 radian
- [ ] 1 circuit
- [ ] 225 circuits

15 Two bodies are moving with a UCM. In order to have the same angluar velocity,

- [ ] They must complete circuits in the same amount of time.
- [ ] They must start at the same point on the circumference.
- [ ] They must have the same radius.

16  The figure represents a body with UCM. This is

- [ ] absolutely impossible.
- [ ] very improbable
- [ ] possible, granted that two different moments have been represented.

17 120 r.p.m. is equal to:

- [ ] 720 r.p.s.
- [ ] 0.5 r.p.s.
- [ ] 2 r.p.s.
- [ ] 2 revolutions per minute

18 A body describes a UCM with a radius of 2m. After having completed a circuit, the space covered is:

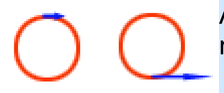
- [ ] 4 m
- [ ] 2 m
- [ ] 0.5 m
- [ ] 2 pi m

19 Three bodies are moving on a segment that is in a motion similar to the one in thevisual. Then:

- [ ] The body that is further away from the center has a greater linear velocity than the other two.
- [ ] The three bodies have the same linear velocity.
- [ ] The body that is nearer to the center always has a greater linear velocity than the other two.

20 two bodies in UCM are moving at different speeds if...

- [ ] They complete a different amount of circuits in the same amount of time.
- [ ] They start at different points.
- [ ] They complete a different amount of circuits.

21  At this moment, the velocity vectors of the movements represented in the figure:

- [ ] are equal.
- [ ] Are in perpendicular directions.
- [ ] Are in opposite directions
- [ ] Are in parallel directions

22 A body is in a Uniform Circular Movement with a radius of 2 m. If it completes one circuit in one minute, its angular velocity in the Système International d`Unités is:

- [ ] 2 pi rad /s
- [ ] 1 r.p.m.
- [ ] pi/30 rad/s
- [ ] 2 m/s

23 3500 r.p.s. is equal to:

- [ ] 35 r.p.m.
- [ ] 210000 r.p.m.
- [ ] 7000 r.p.m.
- [ ] 58.3 r.p.m.

24 (3/2)pi radians are equivalent to:

- [ ] 1.5 circuits.
- [ ] 270 sexagesimal degrees.
- [ ] 360 sexagesimal degrees

25 In its rotation, Earth`s angular velocity is:

- [ ] 24 days per hour
- [ ] 1440 r.p.m.
- [ ] 6.94*10-4 r.p.m.

26 If one body is moving at an angular velocity double that of another body, then

- [ ] It covers half the angular distance.
- [ ] It covers double the angle.
- [ ] It covers double the distance.
- [ ] It completes half the number of circuits.

27 Length of arc can be calculated:

- [ ] Subtracting the number of radians from the radius.
- [ ] Adding the number of radians to the radius.
- [ ] Dividing the number of radians by the radius.
- [ ] Multiplying the number of radians by the radius.

28 A body is in U.C.M. After 3.91 s it has completed 3.98 circuits. Therefore, its speed is:

- [ ] 61.1 r.p.m.
- [ ] 0.016 r.p.m.
- [ ] 0.98 r.p.m.

29 A body describing a Uniform Circular Movement (there is more than one correct answer)

- [ ] Always follows a circular trajectory.
- [ ] Always goes at the same speed.
- [ ] Can follow a straight trajectory
- [ ] Does not describe a trajectory

30 A body describing a UCM with a radius of 1.83 m, after having covered an angle of 6 radians, has covered a space equal to:

- [ ] 10.98 m
- [ ] 6 m
- [ ] 1.83 m
- [ ] 0.305 m

31 Three bodies are moving on a segment that is in a motion similar to the one in the visual **4.2**. Then:

- [ ] The body that is nearer to the center always has a greater angular velocity than the other two.
- [ ] The three bodies have the same angular velocity.
- [ ] The body that is further away from the center has a greater angular velocity than the other two.

32 When the value of control B in the visual **2.1.1** is 4, the speed of the circular movement is:

- [ ] 0.08 r.p.m.
- [ ] 12.64 r.p.m.
- [ ] 25.29 r.p.m.